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Gauge Symmetry and Localized Gravity in M Theory

Nemanja Kaloper[1], Eva Silverstein[1,2], and Leonard Susskind[1]

[1]Department of Physics
Stanford University
Stanford, CA 94305

[2] Stanford Linear Accelerator Center
Stanford University
Stanford, CA 94309

We discuss the possibility of having gravity "localized" in dimension d in a system where gauge bosons propagate in dimension d+1. In such a circumstance—depending on the rate of falloff of the field strengths in d dimensions—one might expect the gauge symmetry in d+1 dimensions to behave like a global symmetry in d dimensions, despite the presence of gravity. Naive extrapolation of warped long-wavelength solutions of general relativity coupled to scalars and gauge fields suggests that such an effect might be possible. However, in some basic realizations of such solutions in M theory, we find that this effect does not persist microscopically. It turns over either to screening or the Higgs mechanism at long distances in the d-dimensional description of the system. We briefly discuss the physics of charged objects in this type of system.

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1. Introduction and Summary

In the study of string dualities and the relation of string theory to field theory, the localization of gauge dynamics (or more general quantum field theory dynamics) to a submanifold of spacetime has been analyzed in many contexts. More recently the possibility of localizing gravity has emerged in the study of cut-off AdS spaces [1]. It is natural to wonder therefore whether it is possible to localize gravity along a d-dimensional slice of spacetime in a system where the gauge fields of some symmetry group G propagate in $d+1$ dimensions.

In the most extreme imaginable versions of such a situation, in a d-dimensional description the symmetry G would appear more like a global symmetry than a gauge symmetry. Field lines would fall off faster than appropriate for a gauge symmetry in d-dimensions. On the other hand the conservation of the associated charge would be guaranteed by the fact that the symmetry was gauged in $d+1$ dimensions. Because gravity is localized to d dimensions, one would then have a global symmetry in the presence of gravity. Since the no-hair theorems for black holes in ordinary d-dimensional effective field theory suggest strongly that such an effect is impossible [2], it is likely that this situation could only occur, if at all, in systems with an infinite number of degrees of freedom in a d-dimensional description. In systems with a holographic description in terms of a low-energy worldvolume quantum field theory on (d-1)-branes, the physics must have a conventional interpretation in d dimensions.

In this paper we will obtain results consistent with this expectation by analyzing various systems in M theory. At the level of low-energy effective field theory in $d+1$ dimensions, the effect appears possible, even generic. Consider a $d+1$-dimensional system involving a gauge theory with field strength F coupled to a scalar ϕ and gravity. The action takes the form

$$S = \int d^d x dr \sqrt{g} \Big(a(\phi) R + b(\phi)(\nabla\phi)^2 + c(\phi) F^2 - \Lambda(\phi) \Big). \tag{1.1}$$

Here a, b, c and Λ are general functions of ϕ. Many such systems have "warped" solutions which give a localized graviton in d dimensions upon integrating over the $d+1$st coordinate r in (1.1). The dimensional reduction of the F^2 term can in general behave differently from that of the Einstein term, since its kinetic term involves an extra power of the inverse metric $g^{\mu\nu}$ relative to that of gravity, and since its coupling to the scalar will in general differ. At the level of (1.1) (without worrying about its embedding into quantum gravity) there

will be a large class of systems for which the coefficient of the F^2 term will diverge. This indicates that the long-distance behavior of the F field is weaker than that of an ordinary unscreened un-Higgsed gauge field in d dimensions.

One such case is the cutoff AdS_5 system with $d = 4$, which we study in §2.[1] In this system, however, the divergence in the F^2 term is logarithmic in the energy scale of the dual conformal field theory coupled to gravity, so that the effect is identical to that of screening of the charge in 4 dimensions.[2] We calculate the electrostatic potential for a test charge at the "Planck brane" and find a result consistent with this interpretation. In realizations of this system in type IIB supergravity, the charged matter that leads to the screening is evident, although the effect arises from geometry independent of assuming the presence of charged matter in the bulk.

We also study the pattern of infrared divergences in kinetic terms for arbitrary q-form gauge potentials in arbitrary dimension. This suggests a "screening" phenomenon for higher-form fields in a range of dimensions.

A more interesting case is a linear dilaton solution of string theory, which we consider in §3. We study in particular the type IIA and type IIB Neveu-Schwarz fivebrane solutions (for which $d = 6$). In this case, the string theory solution again has a diverging F^2 term while the Einstein term survives with a finite coefficient upon dimensional reduction from $7d$ to $6d$. The gauge field propagates as if in $7d$ flat space in the linear dilaton solution. However, the IIA solution gets corrected to one localized along the eleventh dimension of M-theory [5], so that the symmetry is effectively Higgsed. In the IIB case one also finds a lifting of the RR two-form potential from effects occurring in a region where the linear dilaton solution has broken down.

In §4 we discuss some aspects of the physics of charged black holes that make the $d+1$ and d dimensional descriptions of these systems consistent. Finally in §5 we discuss other long-wavelength solutions which naively exhibit this effect and discuss prospects for realizing them in M theory.

One possible application is to the problem of compactifying matrix theory down to four dimensions. This requires considering D0-branes in a IIA compactification down to three dimensions. This is problematic in part because of the infinite classical electrostatic self-energy of the D0-brane in $3d$. If the electric field lives in $4d$, while gravity is localized to $3d$, this problem may be avoided.

[1] Gauge fields in the bulk of the Randall-Sundrum approach to the hierarchy problem [3] were studied by [4].

[2] We thank E. Witten for pointing this out.

2. Cutoff AdS Space

In AdS spaces cut off by a Poincare-invariant "Planck brane", the metric can be written

$$ds^2 = e^{-2|r|/L} dx_{||}^2 + dr^2 \tag{2.1}$$

where $x_{||}$ refers to the dimensions along the brane. The dimensional reduction of the Einstein term in the $5d$ action gives a finite $4d$ Planck scale M_4 [1]:

$$M_4^2 \propto M_5^3 \int_0^\infty dr e^{-2r/L} \tag{2.2}$$

where M_5 is the $5d$ Planck scale. On the other hand, if we introduce a Maxwell field, its kinetic term in $4d$ is divergent. In terms of an infrared cutoff R, the corresponding integral is

$$\frac{1}{e^2} \sim \int_0^R dr \propto R, \tag{2.3}$$

which diverges as $R \to \infty$. From the metric (2.1), this cutoff scale R on the coordinate r corresponds to -Llog $(k_0 L)$ where $k_0 \ll L^{-1}$ is an infrared momentum cutoff along the four dimensions parameterized by $x_{||}$. So this effect is quite conventional in four dimensions: the charge is screened by the charged matter in the $4d$ conformal field theory dual to this background [6].[3]

This result agrees with that obtained by a direct calculation of the electrostatic potential of a charge localized on the Planck brane. (See [4] for similar calculations for gauge fields in [3], and for example [7][8][9] for analogous calculations of corrections to the gravitational propagator.) The electrostatic potential is found by integrating the Maxwell's equation

$$\nabla_M F^{MN} = -QJ^N \tag{2.4}$$

in the background geometry (2.1) in the electrostatic approximation, when the vector potential is $A_M = (\Phi, 0, 0, 0, 0)$ and the density current is $J^M = \frac{1}{\sqrt{g}} \delta^{(4)}(x - x_0)(1, 0, 0, 0, 0)$ (here x stands for all spatial coordinates in (2.1)). To compute the potential, it is convenient to choose the coordinates such that the metric (2.1) is conformally flat. Defining $|z| + L = L\exp(|r|/L)$, the metric becomes

$$ds^2 = \frac{L^2}{(|z| + L)^2} (dx_{||}^2 + dz^2), \tag{2.5}$$

[3] In the realization of AdS_5 in the IIB compactification on S^5, there is a factor of N^2 in the expression for the renormalized $1/e^2$. This comes from the factor $Vol(S^5)/(l_s^8 g_s^2) = N^2/L^3$ in front of the gauge field kinetic term.

while the Maxwell's equations (2.4) reduce to

$$(|z|+L)(\frac{\Phi'}{|z|+L})' + \vec{\nabla}^2\Phi = Q\delta^{(3)}(\vec{x}-\vec{x}_0)\delta(z). \tag{2.6}$$

Since the cutoff AdS_5 space is realized with the orbifold symmetry $r \to -r$, enforcing this symmetry requires $\Phi(z,\vec{x}) = \Phi(|z|,\vec{x})$, and so defining the variable $\rho = 1+|z|/L$ and Fourier transforming in the longitudinal spatial directions, (2.6) becomes

$$\rho^2\frac{d^2\tilde{\Phi}}{d\rho^2} - \rho\frac{d\tilde{\Phi}}{d\rho} - \rho^2\vec{k}^2L^2\tilde{\Phi} = (QL - 2\frac{d\tilde{\Phi}}{d\rho})\delta(\rho-1). \tag{2.7}$$

It is straightforward to see that the only solutions of the homogeneous part of this equation which are regular on the AdS horizon $|z| \to \infty$ are $\tilde{\Phi} = A\rho K_1(kL\rho)$, where $K_n(x)$ are the Macdonald functions of index n (also known as modified Bessel functions of the third kind) and $k = \sqrt{\vec{k}^2}$. The potential is then determined by choosing the integration constant A to satisfy the boundary condition $2\frac{d\tilde{\Phi}}{d\rho}\Big|_{\rho=1} = QL$, as required by the δ-function source in (2.7). The solution is

$$\tilde{\Phi}(\rho,\vec{k}) = -\frac{Q\rho}{2kK_0(kL)}K_1(kL\rho). \tag{2.8}$$

Returning to the original coordinates of the cutoff AdS space (2.1) and Fourier transforming back, we find the electrostatic potential of a particle located on the cutoff brane:

$$\Phi(r,\vec{x}) = -\frac{Q}{2}e^{|r|/L}\int\frac{d^3\vec{k}}{(2\pi)^3}\frac{K_1(kLe^{|r|/L})}{kK_0(kL)}e^{i\vec{k}\cdot(\vec{x}-\vec{x}_0)}. \tag{2.9}$$

To diagnose screening, we consider the potential at very long distances $|\vec{x}-\vec{x}_0| \gg L$ along the cutoff brane $r = 0$. This is dominated by the $k \to 0$ contributions to the integral, where the momentum space potential is

$$\tilde{\Phi}(0,\vec{k}) \to \frac{Q}{2k^2L\ln(kL/2)}, \tag{2.10}$$

which is precisely the potential of a screened charge $Q(k) = \frac{Q}{2L\ln(kL/2)}$, confirming the expectation. We note that in the case of gravity localized to an intersection of n $2+n$-branes in AdS_{3+n+1} [10] a similar analysis shows that the screening effect persists.

In the case of AdS_{d+1} for $d > 4$, the integral determining the gauge coupling is finite. In the case $d = 3$, one also obtains a result consistent with screening behavior: the electrostatic potential goes like $1/x_{||}$ along the brane. This case might be of interest for a matrix theory formulation of $4d$ gravity.

It is instructive to consider the analogous calculations for higher-form gauge potentials. The kinetic energy is (for a gauge field with a q-form field strength)

$$\int d^d x dr \sqrt{g} F_{\mu_1 \ldots \mu_q} g^{\mu_1 \nu_1} \ldots g^{\mu_q \nu_q} F_{\nu_1 \ldots \nu_q}. \tag{2.11}$$

Integrating this over r up to a cutoff R yields an effective charge

$$\frac{1}{e_q^2} \propto \begin{array}{c} e^{\frac{R}{L}(2q-d)} \sim \dfrac{1}{k_0^{2q-d}} \qquad 2q \neq d \\ R \sim log(k_0) \qquad 2q = d \end{array} \tag{2.12}$$

where k_0 is an infrared momentum cutoff along the $x_{||}$ directions. So for $2q \geq d$, the charge is effectively screened at long distance, more strongly for higher-form field strengths. It would be interesting to understand microscopically how this screening occurs–perhaps it arises from spherical $q-2$-branes, which can develop multipole moments as discussed by Myers in [11]. One caveat is that additional interactions can cause the $q-1$-form potential to be lifted at low energies instead. In the next section we will see examples of this possibility.

It is interesting that the effects of light charged matter arise from the AdS part of the gravity solution alone. In known supersymmetric M-theoretic realizations of AdS solutions, there is a five-dimensional Einstein manifold whose isometries yield gauge symmetries and whose Kaluza-Klein excitations provide charged matter. At the level of low-energy field theory, one could contemplate a situation where the gauge field was present in the bulk but charged matter lived only on the brane. In such a situation, the $4d$ behavior of the Maxwell field would be as if there were screening by light charges, but there would be no dynamical charges in the bulk that could be excited.[4] It seems likely that such a situation does not occur in M theory realizations of AdS (this is certainly true in the case of the supersymmetric realizations that are most familiar). In particular, as discussed in §4, bulk charged matter plays a crucial role in black hole physics in these systems.

[4] Something somewhat analogous happens in at the conifold singularity in Calabi-Yau moduli space if the string coupling is taken to zero before the singularity is reached: then the light wrapped D-branes that are usually responsible for the screening of the RR charge in that system [12] are decoupled, and the effect comes from the singular "throat CFT" [13].

3. $6d$ Little String Theories

Consider the N-NS 5-brane solution of type II string theory [14]. It has a string-frame metric and dilaton

$$
\begin{aligned}
& ds^2 = dx_6^2 + dr^2 + l_s^2 N d\Omega_3^2 \\
& \phi = \alpha r
\end{aligned}
\tag{3.1}
$$

with $\alpha = 1/l_s\sqrt{N}$. There is a flux of the three-form NS field strength H which stabilizes the S^3 component of the geometry. We want to consider the behavior of gravitons and of RR gauge potentials in this background.

The decoupled throat theory has this metric with $r \in (-\infty, +\infty)$. For $N \leq 16$ we can cut it off by considering it as part of a compactification, so the full metric is rather complicated but asymptotes to (3.1) down the throat $r \to +\infty$ of the NS5's. This is similar to the proposal [15] for realizing the Randall-Sundrum background in string theory via compactification. The compactification which does this most simply is on the moduli space of type II on $T^4/I_4(-1)^{F_L}$ as studied by Kutasov [16] and by Sen [17]. Note that in the string-realized AdS_{d+1} cases this was not a possibility, since a compactification transverse to the brane would explicitly break the SO(10-d) symmetry of the S^{9-d} surrounding the brane. Here we are not making use of the analogue of that symmetry, but instead are using the U(1) generated by the IIA RR 1-form potential. To consider $N > 16$, we would need a "Planck brane" of the sort considered in [1] in order to cut off the solution and bind gravity. We do not know if this has a precise realization in M theory, but will assume so in discussing this case.

The string coupling grows down the throat of the solution, so most calculations are out of control far down the throat. The corrections to the solution as discussed for example in [5] will be important.

The string-frame ten-dimensional action is

$$
\int d^6x dr d\Omega_3 \left[e^{-2\phi}(R + (\partial\phi)^2) + K_{RR}^2 \right]
\tag{3.2}
$$

where K is the field strength for the RR U(1) gauge field of type IIA string theory, or the field strength for the 2-form RR gauge potential of type IIB string theory.

Consider the perturbation $g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$ in this linear dilaton background, where μ, ν run along the 6 dimensions of the 5-branes. Let us dimensionally reduce the system to $5+1$ dimensions, and write an effective action for the graviton $h_{\mu\nu}$. Though we here work in string frame, the Einstein frame description of course yields the same results.

In determining the 6d Einstein term we must integrate over the extra 4 dimensions in the NS5 background. The contribution of the rest of the compactification is finite, so the only issue is the throat. The integral over r relevant for the $6d$ Planck scale goes like

$$M_6^4 \sim m_s^5 \int dr e^{-2\alpha r} = \frac{m_s^5}{2\alpha} \tag{3.3}$$

which is finite in terms of the string mass scale $m_s = 1/l_s$. (Here for simplicity we have placed the Planck brane at $r = 0$.)

Now do the same for the RR gauge field. It does not couple to the dilaton in string frame (in its realization with a standard gauge invariance, rather than one intertwined with the dilaton) [18]. So the integral over r is infinite. This already shows a definite difference from a standard gauge symmetry: as reviewed above, in the AdS cases this integral is finite for 1-form gauge potentials in $d > 4$, reflecting the fact that the higher-dimensional QFTs do not screen the charge via quantum effects.

Furthermore, Maxwell's equations are easy to solve here, since we are in flat space with no extra direct couplings to ϕ as far as the RR gauge field is concerned. The power law falloff of the electric field will be as in 7 dimensions rather than 6. So it naively looks like this is a case where the gauge field propagates in one higher dimension than gravity, which is "localized" in 6d. We will find, however, that this is not true; the symmetry is Higgsed.

In [19], arguments against global symmetries in perturbative string theories were presented. It should be noted that the effect we see here at the level of perturbative string theory is not in contradiction with the results of [19] for two reasons: (i) perturbation theory breaks down down the throat and (ii) the throat is noncompact (there is a continuum of modes).

3.1. Type IIA

In the type IIA string theory, the strong coupling limit occuring down the throat of the solution is better described by the eleven-dimensional limit of M-theory. In this description, the eleventh circle S^1_{11} is expanding down the throat of the solution. Lifting the solution (3.1) directly to eleven-dimensional supergravity gives a configuration where M5-branes are "smeared" over the eleventh dimension. For a finite number N of NS5-branes, this solution is microscopically corrected to one in which N M5-branes are localized at points on S^1_{11} [5]. For an infinite number of 5-branes, one can contemplate the smeared

solution corresponding to a continuus placement of the infinite number of branes along the circle, as in analogous cases studied in the context of the AdS/CFT correspondence [20]. However, as we will review, the scaling of parameters that leads to this solution invalidates the supergravity approximation and more importantly removes the localization of the graviton.

Microscopics

Starting with finite N, we can find a microscopic embedding of the cutoff solution into M theory. This is obtained by the construction of [16], in which the NS5-branes sit at points in a compactification manifold. The full solution is [5][21]

$$
\begin{aligned}
ds^2 &= l_p^2 f^{-\frac{1}{3}} \left[dx_{||}^2 + f(dy_{11}^2 + dU^2 + U^2 d\Omega_3^2) \right] \\
f &= \sum_{n=-\infty}^{\infty} \sum_{i=1}^{N} \frac{1}{[U^2 + (y_{11} - y_i + \frac{n}{l_s^2})]^{\frac{3}{2}}}
\end{aligned}
\tag{3.4}
$$

where the $y_i, i = 1, \ldots, N$ denote the positions of the N branes on S^1_{11}. The metric is written here in terms of the coordinate $U = \frac{\sqrt{N}}{l_s^2} e^{\frac{r}{\sqrt{N} l_s}}$.

Since this solution breaks the translation symmetry along S^1_{11}, the U(1) RR 1-form potential is Higgsed. This is an infrared effect in the $6d$ description. The microphysics of M theory therefore avoids the issue of a global symmetry arising in $6d$, by substituting spontaneous breaking of the symmetry.

This Higgsing persists in the appropriate $N \to \infty$ limit. In terms of these coordinates (which correspond to canonically normalized VEVs of fields in the (2,0) conformal field theory [21]) the periodicity of y_{11} is $1/l_s^2$. From (3.1) (with $\alpha = \frac{1}{l_s \sqrt{N}}$) it is clear that to localize gravity as $N \to \infty$ we need $l_s \to 0$ so that $l_s \sqrt{N}$ does not diverge in the limit. This is also required for having a valid supergravity approximation everywhere [22]. For evenly-spaced branes, the spacing between branes is fixed:

$$
\Delta y = y_i - y_{i-1} = \frac{1}{l_s^2 N}. \tag{3.5}
$$

So what happens in this limit is that the eleventh circle S^1_{11} expands as $N \to \infty$, leaving the spacing between branes fixed and the symmetry of interest here broken. The smeared solution, which preserves the symmetry, could only pertain to a different scaling which removes the localization of gravity.

3.2. Type IIB

In the type IIB theory, the relevant gauge potential is the RR 2-form B_{RR}. In this case, as one increases r, the solution (3.1) crosses over to the D5-brane solution and then to a description in terms of the SYM theory formed by the light open strings living on the D5-brane [5]. In the D5-brane and SYM descriptions, the original RR B field becomes an NS B field.

In the open string description, the B field couples to the worldvolume $U(1)$ gauge field via the Stuckelberg coupling [23]

$$S = \int d^{10}x|dB|^2 + \int d^6x(B-F)^2 \tag{3.6}$$

The second term effectively gives a mass to the B field. Thus in this case also, the massless gauge boson gets lifted down the throat of the solution instead of leading to a global symmetry in $6d$. This is consistent with the T-duality to the IIA case on a circle.

4. Black Hole Physics

Even with the conventional d-dimensional understanding that we have come to of the physics of $d+1$-dimensional gauge fields in our systems, the $d+1$-dimensional picture raises interesting questions about black hole physics. We will here provide a qualitative discussion of some of these issues; it would be interesting to find concrete black hole solutions in these backgrounds to study. Schwarzschild black holes were considered in [9]; in the systems we are considering here the generalization to charged black holes is of interest.

In the AdS cases, the symmetry is unbroken and the charge is conserved (and screened in low enough dimension). Suppose there is an extreme or nearly extreme black hole of charge Q and mass M centered on the Planck brane in AdS_5. Its charge can be measured by Gauss' law in five dimensions, and this charge is conserved overall in the system. In the four-dimensional description, the charge is screened, and could not be measured at long distance. One expects quantum mechanically a charged black hole to be quickly neutralized at long distances by the light charged conformal field theory matter in the system (in analogy to the familiar effect in ordinary QED in black hole backgrounds [24]).

How does this occur from the five-dimensional point of view? In order to see the neutralization from this point of view, we need the electric field to be strong enough to make it energetically favorable for charged matter to be pair-produced and to draw

the negatively charged member of the pair to the black hole (against the gravitational attraction toward the AdS horizon). Let us assume we have charged matter of mass m and charge q in the bulk theory. From (2.9) for large $|z|$ one finds

$$F^{0z} \quad \propto \quad \frac{Q|z|^2}{L^5} \tag{4.1}$$

So the strength of the electric field grows toward the AdS horizon. Since this will eventually dominate over the mass, we expect an analogue of the Schwinger calculation to imply pair production (though we have not calculated this effect in our background directly). In terms of the forces on the produced pair, the source Q leads to an electromagnetic force on the particle of charge q in the z direction of magnitude

$$f^z_{E.M.} \quad \propto \quad \frac{qQz}{L^4}. \tag{4.2}$$

Here we took the particle to be at rest and evaluated the relation $f^\mu_{E.M.} = qF^\mu{}_\nu \frac{dx^\nu}{d\tau}$ in our background field configuration (2.5)(4.1). There is also a gravitational attraction to the black hole, as well as a gravitational attraction toward the AdS horizon. The latter effect goes like

$$f^z_{AdS} \quad \propto \quad -m\Gamma^z_{\mu\nu}\frac{dx^\nu}{d\tau}\frac{dx^\mu}{d\tau} \sim -mz\frac{1}{L^2} \tag{4.3}$$

From (4.2) and (4.3) it is clear that for large enough Q, the electromagnetic force will dominate. We therefore expect dielectric breakdown from pair production of the charged matter of mass m to become possible at large enough $|z|$. For small Q (and small size relative to the AdS radius), the object does not constitute a black hole from the $4d$ point of view in any case.

Another effect to consider is the quantum stability of the localization of the charge Q on the brane. Particles in the bulk fall toward the AdS horizon, and therefore it seems clear that the charged source will ultimately tunnel into the bulk and fall down the throat. In the absence of a genuine embedding of the Planck brane into M theory, we cannot calculate the rate for this tunneling process. To a $4d$ observer it would look like the conserved charge is spreading out. The boundary of this region of spreading charge may behave like a membrane, and the $5d$ gravity description might provide a way to study membrane nucleation processes.

One can similarly consider charged objects in the IIA system of §3 in the regime where the "throat" solution (3.1) applies. The D0-branes in this background (which are momentum modes around S^1_{11}) become light down the throat, exponentially in r:

$$M_{D0} = \frac{1}{e^{\phi} l_s} \sim \frac{1}{l_s} e^{-2\alpha r} \tag{4.4}$$

On the other hand, the electric field emanating from a source of charge $Q > 0$ at the Planck brane decays like a power:

$$\vec{E} \sim \frac{Q}{(r^2 + x_{||}^2)^{\frac{5}{2}}}. \tag{4.5}$$

We again expect that for large enough r, the light D0-branes will be pair-produced and neutralize a charged black hole in this system as well.

There is another intriguing aspect to the physics of charged objects in this sort of system. In $d+1$ dimensions one can measure the charge classically using Gauss' law. The charge that one measures this way is the bare unscreened charge Q. This procedure must translate into some operation in the d-dimensional description of the system. According to the IR/UV relationship in holography [25], the $d+1$-dimensional Gauss' law measurement always uses information that is longer-wavelength than the size of the object in the d-dimensional description.

5. More General Solutions

We have seen that in cases where there is a brane interpretation of a warped metric, M theory conspires to prevent a truly higher-dimensional gauge field from arising in a d-dimensional gravity theory. We expect it is likely that this happens rather generically. Still, it is interesting to consider backgrounds which, like those discussed here, have such an effect naively in a long-wavelength analysis—but whose microscopic behavior is not yet understood.

The linear dilaton solution (3.1) arises much more generally than in the NS5-brane solutions (in general with the $d\Omega_3^2$ piece replaced with something else). Geometrical singularities such as the conifold singularity routinely resolve into a throat with linear dilaton behavior. This can be seen from the description of such compactifications using the techniques of [13]. Some of these cases in fact descend from those considered above via K3

fibration. It would be very interesting to systematically analyze the behavior of gauge bosons in many other classically singular geometries.

Another place where linear dilaton solutions arise is in noncritical perturbative string backgrounds with a tree-level cosmological term. There the linear dilaton solution (3.1) exists with $\alpha^2 \propto (D - D_{crit})/l_s^2$. In this case, as before, a naive calculation would suggest the potential for a delocalized gauge boson. Since the dilaton grows along the direction r in the solution, strong coupling arises and corrections will be important. With current technology we cannot say whether this will always lead to conventional $4d$ behavior or whether it is conceivable that sometimes $4d$ effective field theory will break down in such a way that a global symmetry of the kind we have been contemplating can persist.

One could similarly consider the dilaton gravity solutions of the type considered in for example [26]. As we discussed in the introduction, the coupling of the scalar to the F^2 term can (quite generically) be such that again a naive calculation of the coupling would suggest a globalization of the symmetry in $4d$. However, as above, corrections will be important (and will rule out some subset of these solutions altogether).

The gravity backgrounds we considered in the bulk of the paper have known holographic descriptions in terms of a brane worldvolume theory which reduces to ordinary d-dimensional effective quantum field theory at low energy. Most backgrounds have no known holographic description (for some relatively recent considerations of the general features required see for example [27]), and it is not yet clear what precise form holography will take in a generic background. It will be interesting to study the structure of gauge symmetries in various dimensions once more general backgrounds are understood.

In any case, as it stands, in this paper we have gathered further evidence for the robustness of the arguments against global symmetries in the context of gravity.

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